Modelling of resonant wireless power transfer with integral formulations in heterogeneous media

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This paper deals with the electromagnetic simulation of magnetically coupled wireless power transfer. A recently proposed approach based on a full-wave integral formulation (applicable in homogeneous media only) is extended to the case of heterogeneous media to some extent. In this new integral formulation, a quasi-static approximation is introduced and the effect of a planar boundary between different dielectric half-spaces is treated by the method of images. The results show a performance as expected. The modelling of single coils and full WPT chains is possible.

***Index Terms*—wireless power transfer, resonance, integral equation, method of images**

I. Introduction

Wireless power transfer (WPT) technologies offer the experience of "ubiquitous power" in many areas of our life. Few people realize that there is always a sophisticated design inherent in this technology, a part of which is the simulation of electromagnetic fields playing the key role in the power transfer. In several applications the transfer must be established through lossy dielectric materials such as house walls (wood, concrete, glass, etc.), water or human bodies [1]. It this case polarization and conduction mechanisms may affect the transfer parameters like resonant frequency, quality factors and, after all, transfer efficiency. The preliminary knowledge of these effects is very important in the design and operation of such WPT systems.

The most typical applications of this kind are implantable medical devices with considerable power consumption, and therefore supplied from outside of the body, preferably in a wireless manner. The WPT unit usually consists of two resonator coils, one of which is located inside the human body. The "in-body" and "on-body" coils are separated in the range of centimeters, and this relatively narrow gap between them is an inhomogeneous region formed by dispersive biological tissues (skin, fat, muscle) and air. In [2] such a system is modelled by an equivalent circuit, the lumped parameters of which are extracted by means of finite element computations using time-harmonic quasi-static approximation.

Recently we proposed a computational method based on full-wave integral equations and the method of moments (MoM), by which the WPT system —as a whole— can be simulated and optimized very efficiently [3]. Moreover in [4] we demonstrated that full-wave modelling provides better accuracy and reveals more aspects of the transfer than the more commonly used circuit models based on parameter extraction. However, the method proposed in [3] is limited to homogeneous media. In the present work, this method is extended to heterogeneous media to some extent.

The special geometry of the targeted application is exploited: the two coils are separated by a narrow gap and a (locally) planar material interface. At some applied frequencies (for instance, 1 GHz is proposed in [5]) the coil sizes and the distance between them are electrically short, while the wire length of the coils is still comparable with the wavelength. To handle such specific cases we propose herein a hybrid integral formulation with the quasi-static and full-wave approach combined, and by utilizing the classical method of images.

II. The electromagnetic models

A. Full-wave model in brief

In [3] the coil wire of radius a is represented by a 1-dimensional curve lying on the mid-line of the wire. The coordinate ζ gives the position along this curve. Four unknown, ζ-dependent scalar quantities are then introduced: the current $I(\zeta)$, the charge density $q(\zeta)$, the electric scalar potential $\Phi(\zeta)$ and the ζ-component of the magnetic vector potential $A_\zeta(\zeta)$, respectively. The potentials are expressed in integral forms of the current (1) and the charge (2), taking into account the retardation of the field. In air, we have

$$A_\zeta(\zeta) = \hat{\mathbf{e}}_\zeta \cdot \frac{\mu_0}{4\pi} \int_0^l I(\zeta') \frac{\exp(-\mathrm{j}\beta\chi(\zeta,\zeta'))}{\chi(\zeta,\zeta')} \mathrm{d}\boldsymbol{\zeta}', \quad (1)$$

$$\Phi(\zeta) = \frac{1}{4\pi\varepsilon_0} \int_0^l q(\zeta') \frac{\exp(-\mathrm{j}\beta\chi(\zeta,\zeta'))}{\chi(\zeta,\zeta')} \mathrm{d}\zeta', \quad (2)$$

where the function $\chi(\zeta,\zeta')$ gives the distance between the points of the curve at ζ and ζ' and $\beta = \omega\sqrt{\mu_0\varepsilon_0}$ is the phase coefficient. The integration path is all along the wire of length l. By completing (1)-(2) with the Ohm's law along the wire (3) and the charge conservation law (4) as

$$rI(\zeta) = -\frac{\mathrm{d}\Phi}{\mathrm{d}\zeta} - \mathrm{j}\omega A_\zeta, \quad (3)$$

$$0 = \mathrm{j}\omega q(\zeta) + \frac{\mathrm{d}I(\zeta)}{\mathrm{d}\zeta}, \quad (4)$$

with r being the resistance of the wire per unit length, a system of integro-differential equations is obtained which is solved by the MoM. This model –detailed in [3]– provides accurate results for all frequencies provided that the wavelength is

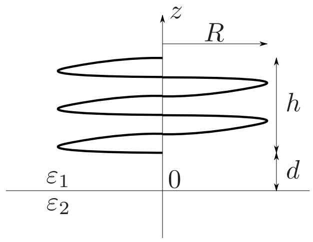

Fig. 1. Scheme of the studied helical coil. The coil's axis is perpendicular to the planar boundary of the half-spaces with homogeneous dielectrics.

much larger than the wire radius. However, only homogeneous dielectric can be treated.

B. Quasi-static integral formulations

In this paper a *quasi-static integral formulation* is proposed. The approximation consists in neglecting the retardation of the fields, i.e., in the expressions (1) and (2), $\beta = 0$ is substituted. This approximation is valid for wavelengths much larger than the coil dimensions. Let us note that the wire length l is usually comparable with the wavelength, i.e., the current and charge varies with ζ. This quasi-static model is thus similar to the transmission line models where the EM field is modelled as a wave in the longitudinal direction but assumed to be quasi-static in the transverse plane.

A benefit of this approach is that some kind of *heterogeneous media* (of practical interest) can be considered: if a planar boundary separates two homogeneous half-spaces with different dielectrics, the method of images [6] can be applied. The space is magnetically homogeneous. Let the $z = 0$ plane divide the space into two half-spaces filled with dielectrics having permittivities ε_1 ($z > 0$) and ε_2 ($z < 0$). Let the coil be embedded in the $z > 0$ half-space. Then the formula (2) of the electric scalar potential in the quasi-static model becomes

$$\Phi(\zeta) = \frac{1}{4\pi\varepsilon_1} \int_0^l q(\zeta') \left(\frac{1}{\chi(\zeta, \zeta')} + \frac{k}{\psi(\zeta, \zeta')} \right) \mathrm{d}\zeta', \qquad (5)$$

where the new function ψ gives the distance between the wire segment at ζ and the *image* of the segment ζ' to the planar boundary and $k = (\varepsilon_1 - \varepsilon_2)/(\varepsilon_1 + \varepsilon_2)$, respectively. All other equations (1) (with $\beta = 0$), (4) and (3), remain unchanged.

III. Numerical example

Herein a study on a single coil (that could be part of a resonant WPT system but not specifically in biomedical applications) illustrates the method, according to the sketch in Fig. 1. The permittivities are $\varepsilon_1 = \varepsilon_0 = 8.854 \times 10^{-12}$ F/m and $\varepsilon_2 = 81\varepsilon_0$, respectively. Geometrical data: $R = 30\,\mathrm{cm}$, $h = 20\,\mathrm{cm}$, $d = 5\,\mathrm{cm}$, the wire radius is $a = 3\,\mathrm{mm}$ and the coil has 5.25 turns. In the MoM-simulation, the discretised wire has 400 segments.

First, the numerical studies (not presented herein) proved that there is only a very small discrepancy between the results of the full-wave and the quasi-static models in the studied frequency band in free-space. Let us note that the wavelength in air at $30\,\mathrm{MHz}$ is $10\,\mathrm{m}$ which is comparable with the wire length

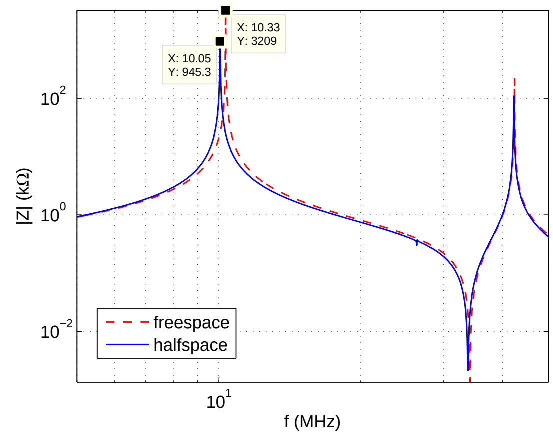

Fig. 2. Magnitude of the coil impedance vs. frequency in the band $5 \ldots 50$ MHz in case of free-space (ε_1) and the half-space, both computed with the quasi-static model.

($\simeq 10\,\mathrm{m}$) but not with the coil dimensions. Second, the quasi-static model has been applied, and two setups are modelled: (i) the whole space is filled by homogeneous dielectric (ε_1), i.e., the free-space case; (ii) the space is heterogeneous as sketched in Fig. 1. The results are presented in Fig. 2. As expected, the resonance peaks slightly shifted downwards as the high-permittivity half-space appears near the coil.

IV. Conclusion, perspectives

The results show the applicability of the proposed method to model the EM field of a single coil near a dielectric boundary. In the full version of the paper, the results of the simulation of full WPT chains in such heterogeneous media will also be presented, with special emphasis on the case where the transmitter and receiver coils are in different dielectrics. An experimental validation of the simulation will be given as well. Considerations on the case of lossy media will also be made, respectively.

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